Stat 582, Spring 2015

Mathematical Probability 2

The purpose if this course is threefold: First, it can be considered as application of basic concepts introduced in Stat 581, such as conditioning, convergence and integration. Second, it introduces new constructs such martingales, random nonnegative integer-valued measures (i.e., point processes) and operator semigroups. Third, it provides examples of how the theory connects with applications. In this latter, it is idiosyncratic, since applications are mainly in genetics and population biology. No biology prerequisite is needed.

No single book will be followed. Martingales are based mostly on Resnick and Williams books, Point processes on Daley and Vere-Jones book, branching and genetic processes on Kimmel and Axelrod and Bobrowski and Kimmeland books. There will be 3-4 homeworks, and a final. At the end of semester, students will report on selected papers they will be assigned as independent study. Lectures will be supplemented by problem-solving sessions, led by the TA.

Topics

- Martingales with applications
  - Refresher on conditional expectations
  - Definitions and examples of martingales
  - Martingales versus submartingales
  - Positive supermartingales
  - Examples (random walks, branching processes, Cannings model in genetics)
  - Martingale and submartingale convergence with applications to bp models

- Diffusions
  - Introduction to Brownian motion
  - Diffusion equation
  - Diffusion approximations
  - Wright-Fisher model in genetics
  - Branching processes

- Point processes with applications
  - Definitions and constructions based on fi-di rectangles
  - Moments and probability generating functionals
  - Clustering and convergence
  - Application to branching processes

- Semigroups of operators
  - Definition, strongly continuous semigroups and their generators
  - Markov semigroups
  - Operator semigroup in population genetics

- Extensions